a) The cost of Running the pipeline strictly on BLM ground with two different cases:


The cost of building the pipeline from west, south, and then east only on the BLM ground will cost $\$ 19,250,000$ and covering 55 miles of ground.

ii) Heading east through the mountain and then south to the refinery.

One time cost: \$4,500,000
Study: \$180,000
5 Month Delay: $5 \times \$ 75,000=\$ 375,000$
$C=\$ 4,500,000+\$ 180,000+\$ 375,000+(24 \times \$ 350,000)+(15 \times \$ 350,000)$
C $=\$ 5,055,000+\$ 8,400,000+\$ 5,250,000$
$C=\$ 18,705,000$
Drilling through the mountain with the 5 month delay for the study will cost $\$ 18,705,000$ and will cover 39 miles of ground.
b) The cost of running the pipeline through the private ground:
i) The shortest distance across the private ground to the refinery.


Private Ground: Extra $\$ 90,000$ per mile

$$
\begin{array}{ll}
\mathrm{a}=15 \text { miles } & \mathrm{b}=24 \text { miles } \\
c^{2}=a^{2}+b^{2} & c^{2}=15^{2}+24^{2} \\
c=\sqrt{801} & c=3 \sqrt{89} \\
C=(\$ 90,000+\$ 350,000) \times 3 \sqrt{89} \\
\mathrm{C}=\$ 12,452,855.09
\end{array}
$$

To cut through the Private Ground it would cost $\$ 12,452,855.09$ and will cover around 28.30 miles of ground.
ii) Straight south across the private ground, then straight east to the refinery.


$$
\begin{aligned}
& \text { Private }+ \text { BLM }=\$ 440,000 \\
& \text { South: } 15 \text { miles } \quad \text { East: } 24 \text { miles } \\
& C=(15 \times \$ 440,000)+(24 \times \$ 350,000) \\
& C=\$ 6,600,000+\$ 8,400,000 \\
& C=\$ 15,000,000
\end{aligned}
$$

The total cost of going south on the private ground and east on the BLM ground is $\$ 15,000,000$ and covering 39 miles.
c) The cost function for the pipeline for the configuration involving running from the well across the private ground at some angle and intersecting the BLM ground to the south and then running east to the refinery. Determine the length of pipe that runs across the private land and how far from the refinery it starts running on BLM land. Determine the angle at which your optimal path leaves the well. This function is used to find the optimal way to fun the pipeline to minimize cost.


$$
\begin{aligned}
& \tan (\theta)=\frac{o p p}{a d j}=\frac{x}{15} \quad y^{2}=x^{2}+15^{2} \quad y=\sqrt{x^{2}+225} \\
& C(x)=\$ 440,000\left(\sqrt{x^{2}+225}\right)+\$ 350,000(24-x) \\
& C(x)=\$ 8,400,000-\$ 350,000 x+\$ 440,000 \sqrt{x^{2}+225} \\
& C^{\prime}(x)=-\$ 350,000+\frac{1}{2}(\$ 440,000) \times\left(x^{2}+225\right)^{-\frac{1}{2}} \times 2 x \\
& C^{\prime}(x)=-\$ 350,000+\frac{8440,000}{\sqrt{x^{2}+225}} \\
& C^{\prime}(x)=\$-350,000\left(x^{2}+225\right)^{\frac{1}{2}}+\$ 440,000 x \\
& 0=-\$ 350,000\left(x^{2}+225\right)^{\frac{1}{2}}+\$ 440,000 x \\
& \$ 440,000=\$ 350,000\left(x^{2}+225\right)^{\frac{1}{2}} \\
& \frac{44}{35} x=\left(x^{2}+225\right)^{\frac{1}{2}} \\
& \frac{1936}{1225} x^{2}=x^{2}+225 \\
& 711 x^{2}=275,625 \\
& x^{2}=\frac{30,625}{79} \\
& x=\frac{175}{\sqrt{79}} \\
& \tan (\theta)=\frac{175}{\sqrt{79}}=\tan (\theta) \approx 1.312602551 \\
& y=\sqrt{\left(\frac{175}{\sqrt{79}}\right)^{2}+225}=24.742 \\
& d=24-\frac{175}{\sqrt{79}}=4.31096 \\
& \tan -1 \\
& \$ 440,000(24.752)+\$ 350,000(4.31096)=\$ 12,399,716 \\
& \\
& \hline
\end{aligned}
$$

The cost of the optimal plan is $\$ 12,399,716$ and the angle it leaves the well is 52.70 degrees. And around 29.06 miles of ground will be covered.
d) Include a computer generated graph of the optimal cost function " $\mathrm{C}(\mathrm{x})$ ", for the pipeline for any configuration involving crossing some private ground as well as some BLM ground. Make sure to use the correct domain of the function to scale your axes appropriately and to label the minimum cost.

$$
C(x)=\$ 440,000\left(\sqrt{x^{2}+225}\right)+\$ 350,000(24-x)
$$



## Refection

I have learned a great deal in my calculus class. Our whole society involves calculus and other mathematics. For example the mean value theorem. If police officers knew about the mean value theorem they could actually prove that a person was speeding. I also enjoyed the angioplasty problem. We were able to find the rate we can fill the catheter in a safely and a timely matter. I work in the Operating Room and I am sure there are problems that can be solved using calculus. Calculus is the math that is used for real world problems and they're not just generic text book problems.

